An Adaptive Hybrid Morphological Method for Designing Translation Invariant Morphological Operators

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Abstract—In this paper an adaptive hybrid morphological method is presented for designing translation invariant morphological operators via Matheron Decomposition and via Banon and Barrera Decomposition. It consists of a hybrid model composed of a Modular Morphological Neural Network (MMNN) and an evolutionary search mechanism: the adaptive Genetic Algorithm (GA) (adaptive rates in genetic operators). The proposed method searches for initial weights, architecture and number of modules in the MMNN; then each element of the adaptive GA population is trained via the Back Propagation (BP) algorithm. Also, it is presented and experimental investigation with the proposed method using a relevant application in image processing, the restoration of noisy images, where it verifies that the method proposed herein allows seamless and efficient design of translation invariant morphological operators of either increasing or non-increasing types, and the results are discussed and compared, in terms of Noise to Signal Ratio (NSR), with the previously methods proposed in literature, showing the robustness of the proposed method.

Keywords—Translation Invariant Morphological Operators, Morphological Neural Networks, Evolutionary Computation, Hybrid Systems, Image Restoration.

I. INTRODUCTION

Mathematical Morphology (MM) represents an important class in image processing. It uses a nonlinear focus based on geometric analysis through using structuring elements, which are small patterns that operates in spatial domain and extract information about the geometric forms in the image [1], [2]. The MM was created around 1960, being developed by Matheron [3] and Serra [4] with goal of processing boolean images. Sternberg [5] and Serra [4] extended such concept for gray scale image processing. The MM formal definition, in terms of lattice algebra, was presented by Heijmans [6]. An important result for MM was presented by Banon and Barrera [7], the decomposition theorem of translation invariant operators, which guarantees that every translation invariant operator, not necessary increasing, may be decomposed by a combination of basic operators: dilation, erosion, anti-dilation and anti-erosion. The Banon and Barrera [7] theorem result is a generalization of the Matheron canonic decomposition theorem [3] for increasing translation invariant operators.

The design of translation invariant operators is a relevant problem in mathematical morphology, with applications in image processing, like image restoration, edge extraction and object recognition. Many works have focused on the design of this kind of operators. Yang and Maragos [8] designed operators (min-max classifiers) according to Matheron decomposition [3] by using the mean square error for minimizing the cost function. Dougherty and Loe [9] designed sub-optimal operators satisfying Matheron theorem [3] for binary image processing. Davidson and Hummer [10] used Morphological Neural Networks (MNNs) for designing morphological filters, differing from the classical neural networks [11] in the sense that the computation in each node of the MNN is carried out by simple morphological operators in the context of Algebra of Images [12]. Herwing and Shalkoff [13] presented a MNN with learning based on the delta rule for designing filter for binary images.


This paper presents an adaptive hybrid morphological method for designing translation invariant operators via Matheron Decomposition and via Banon and Barrera Decomposition for image restoration of images corrupted by salt and pepper noise. It consists of a hybrid model composed of an MMNN [1], [2] and an adaptive Genetic Algorithm (GA) [18], which may be regarded as an improvement of the methodology described in [17] as follows: (a) the GA is adaptive for faster convergence; (b) each element of the adaptive GA population represents an MMNN; and (c) an adaptive GA is used to search for the initial weights, architecture and number of modules.
of the MMNN, wherein each element of the adaptive GA population is trained via the BP algorithm.

II. BACKGROUND

A. Mathematical Morphology

Mathematical Morphology represents an important class of nonlinear signal processing systems, which aims to quantitatively describe the geometrical structure of a signal using structuring elements. The following equations are used in MMNN for designing translation invariant operators [1], [2]:

\[
\text{Dilation: } \delta_k = \max \left( x + \bar{a}_k \right);
\]

\[
\text{Erosion: } \epsilon_k = \min \left( x - \bar{a}_k \right);
\]

\[
\text{Anti-Dilation: } \delta_k^c = 1 - \min \left( x - \bar{b}_k \right);
\]

\[
\text{Anti-Erosion: } \epsilon_k^c = 1 - \max \left( x + \bar{b}_k \right),
\]

where \(x\) is the input signal and \(\bar{a}_k\) and \(\bar{b}_k\) represent the structuring element (terms \(\bar{b}_k\) represent the reflection of the complement of the structuring elements of anti-dilation or anti-erosion).

B. Adaptive Genetic Algorithm

The adaptive GA of Mitsuho and Cheng [18] differs from SGA by using adaptive methods applied to crossover and mutation operators. The method adopted in the present work is the deterministic adaption [18] and consists in modifying the operators rate according to a pre-stablished rule. The operators rates are gradually decreased in each population evolution. The following equation defines the rule adopted as the adaptive parameter in the rates of crossover and mutation:

\[
T_{x_a} = T_{x_i} - (T_{x_i} - T_{x_f}) \ast \frac{g_a}{G},
\]

where \(T_{x_a}\), \(T_{x_i}\), and \(T_{x_f}\) represent the current, initial and final rates. The terms \(G\) and \(g_a\) represent, respectively, the maximum number of generations and the current generation.

Figure 1 illustrates the adaptive GA procedure.

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According to Figure 1, the adaptive GA procedure starts with the creation of an individuals’ population, or more specifically, the solutions set. Then, each individual is evaluated by a fitness function (or cost function), which is an heuristic function that guides the search for an optimal solution in state space. After evaluating the GA population, it’s necessary to use some procedure to select the individual parent pairs, which will be used to perform the genetic operators (crossover and mutation). The next step is responsible for performing the crossover genetic operator. Usually, the crossover operator mixture the parents genes for exchanging genetic information from both, obtaining its offspring individuals. After crossover operator, all offspring individuals will be the new population, which contains relevant characteristics of all individuals parent pairs obtained in selection process. The next step is to mutate the new population. The mutation operator is responsible by the individual genes aleatory modification, allowing the population diversification and enabling GA to escape of local minima (or maxima) of the surface of the cost function (fitness). Finally, the new mutated population is evaluated and then the crossover and mutation rates are updated according to Equation 5. This procedure is repeated until a stop condition be reached.

III. MMNN FUNDAMENTALS

A. MMNN Architecture for the Matheron Decomposition

Sousa [1], [2] defined the MMNN for designing translation invariant operators that satisfy the Matheron decomposition theorem [3] for dilations as well as for erosions. The Matheron theorem [3] states that every increasing and translation invariant operator may be decomposed by a union of erosions or a intersection of dilations. Figure 2 presents the MMNN architecture for the Matheron decomposition [3] by dilations. The following equations define the MMNN architecture for the Matheron decomposition [3] via dilations according to this approach.

\[
v_k = \delta_k = \max \left( x + \bar{a}_k \right),
\]

where \(x\) is the input signal of the MMNN.

Network output: \(Y = \min (\bar{v})\),

where \(\bar{v} = (v_1, v_2, \ldots, v_k)\).

The weight matrix, \(A\), of the MMNN is defined by

\[
A = (\bar{a}_1; \bar{a}_2; \ldots; \bar{a}_k),
\]

where \(\bar{a}_k \in \mathcal{R}^k, k = 1, 2, \ldots, N\), represent the MMNN weights (i.e. rows composed by structuring elements \(\bar{a}_k\)). Symbol \(\Lambda\) represents the minimum operation.

In a dual manner, the MMNN architecture for the Matheron decomposition [3] via erosions is defined by substituting dilations by erosions and symbol \(\Lambda\) by \(\lor\), where \(\lor\) represents the maximum operation. Figure 3 presents the MMNN architecture for the Matheron decomposition [3] by erosions.
\[
\begin{align*}
A(k+1) &= A(n) - \mu \nabla_A J(A), \quad n = 0, 1, \ldots \\
\text{where} \quad \mu &= \text{the learning rate and } \nabla_A J(A) = \text{gradient matrix for some objective function } J(A) \text{ (to be minimized with respect to the weight matrix } A). \quad (18)
\end{align*}
\]
The gradients presented in equations (23) and (24) are given by
\[
\frac{\partial J}{\partial a_k} = -\epsilon \frac{\partial y}{\partial v_k} \frac{\partial v_k}{\partial a_k}, \quad k = 1, 2, \ldots, N. \tag{26}
\]
\[
\frac{\partial J}{\partial b_k} = -\epsilon \frac{\partial y}{\partial v_k} \frac{\partial v_k}{\partial b_k}, \quad k = 1, 2, \ldots, N. \tag{27}
\]
According to Sousa [1], [2], the partial derivatives in equations (26) and (27) are estimated by the methodology of Pessoa and Maragos [19] via rank indication vectors \( c \) and smooth impulse functions \( Q_\sigma \), and are given by
\[
\nabla_A J(A, B) = -\epsilon \text{diag}(\bar{c}) \text{diag}(\bar{c}_1) C_1, \tag{28}
\]
\[
\nabla_B J(A, B) = -\epsilon \text{diag}(\bar{c}) \text{diag}(\bar{c}_2) C_2, \tag{29}
\]
where
\[
\bar{c}_1 = (\bar{c}_{11}, \bar{c}_{21}, \ldots, \bar{c}_{N1}), \quad \bar{c}_2 = (\bar{c}_{12}, \bar{c}_{22}, \ldots, \bar{c}_{N2}),
\]
\[
(\bar{c}_1, \bar{c}_2) = \frac{Q_{\sigma}(\min(u_k), 1-u_k)}{Q_{\sigma}(\min(u_k), 1-u_k), 1}.
\]
\[
C_1 = (\bar{c}_{11}; \bar{c}_{21}; \ldots; \bar{c}_{N1}), \quad \bar{c}_{1k} = -\frac{Q_{\sigma}(u_k, 1-x-u_k)}{Q_{\sigma}(u_k, 1-x-u_k), 1},
\]
\[
C_2 = (\bar{c}_{12}; \bar{c}_{22}; \ldots; \bar{c}_{N2}), \quad \bar{c}_{2k} = -\frac{Q_{\sigma}(1-u_k, 1-x-u_k)}{Q_{\sigma}(1-u_k, 1-x-u_k), 1}.
\]

V. THE PROPOSED METHOD

The method proposed in this paper, referred to as adaptive hybrid morphological operators design (AHMOD) method, uses an adaptive evolutionary search mechanism for designing translation invariant operators via the Matheron decomposition [3] and the Banon and Barrera decomposition [7] for image restoration of images corrupted by salt and pepper noise. It consists of a hybrid model composed of an MMNN [1], [2] and an adaptive GA [18], which searches for the initial weights, architecture and number of modules of the MMNN, wherein each element of the adaptive GA population is trained via the BP algorithm. Each element of adaptive GA population represents an MMNN. As an example, Figure 6 represents an element of the adaptive GA population, where \( s(e_i), i = 1, 2, \ldots, N \), denotes the initial MMNN weights. The term \( \text{arch} \) indicates the MMNN architecture (each architecture has a corresponding training algorithm). The term \( \text{mod} \) represents the number of MMNN modules. Table I presents an example of coding used for identifying the MMNN architectures. Figure 7 illustrates the general scheme of the proposed method.

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Fig. 5. MMNN architecture used for the Banon and Barrera decomposition via inf-generators.

Fig. 6. Coding of the adaptive GA chromosome.
Fig. 7. General scheme of the proposed method.

TABLE I
EXAMPLE OF CODE FOR THE MMNN ARCHITECTURES.

<table>
<thead>
<tr>
<th>Code</th>
<th>MMNN architecture</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Matheson decomposition via dilations</td>
</tr>
<tr>
<td>1</td>
<td>Matheson decomposition via erisions</td>
</tr>
<tr>
<td>2</td>
<td>Banon and Barrera decomposition via sup-</td>
</tr>
<tr>
<td></td>
<td>generators</td>
</tr>
<tr>
<td>3</td>
<td>Banon and Barrera decomposition via inf-</td>
</tr>
<tr>
<td></td>
<td>generators</td>
</tr>
</tbody>
</table>

VI. SIMULATIONS AND RESULTS

The noise to signal ratio (NSR) is used for assessing performance of the designed operator when applied to restoration of images. It is defined by

\[ NSR = 10 \log_{10} \frac{(D - Y)^2}{(D)^2}, \]

where \((D - Y)^2\) and \((D)^2\) represent the mean energy of the error (second moment of the error) and the mean energy of the desired output (second moment of the target).

The MMNN is trained via an adaptive GA with initial population of 100 elements, maximum of 100 generations, with an interval of adaptive variation \(T_x_t = 1.0\) to \(T_x_f = 0.5\) for crossover probability and \(T_x_t = 0.05\) to \(T_x_f = 0.0001\) for mutation probability, according to [18]. The adaptive GA stopping criterion is the number of iterations of the GA. Each element of the adaptive GA population is then trained via the BP algorithm for 100 epochs, using smooth rank function \((Q_\sigma = \exp \left( -\sigma^2 \right) / 2 \right) \) with the smoothing parameter \(\sigma = 0.05\), and a convergence factor \(\mu = 0.01\). The training set consists of 25% of a given image (that represents a continuous region of the image), whereas the test set consists of the entire image. All images are normalized within the range \([0,1]\); grayscale structuring elements are normalized in the range \([-1,1]\).

A. Applications

A classical problem in image processing is restoration of images corrupted by noise [20]. The present paper considers salt and pepper noise. The classical median filter [20] is an alternative commonly used for restoring images corrupted by that noise.

The Table II presents the results obtained by the proposed method (AHMOD Method) and by the methods proposed in literature, where MEDIAN denotes the median filter [20] result, MMNN (SGA) and MMNN (BP) denote the results obtained by Sousa [1], [2] and MMNN (AGA) and MMNN (AGA-MOD) denote the results obtained by Araújo et al. [17].

<table>
<thead>
<tr>
<th>Method</th>
<th>MMNN mod</th>
<th>MMNN arch</th>
<th>NSR (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEDIAN</td>
<td>-</td>
<td>-</td>
<td>-21.55</td>
</tr>
<tr>
<td>MMNN (SGA)</td>
<td>8</td>
<td>1</td>
<td>-17.63</td>
</tr>
<tr>
<td>MMNN (SGA)</td>
<td>8</td>
<td>0</td>
<td>-19.00</td>
</tr>
<tr>
<td>MMNN (SGA)</td>
<td>25</td>
<td>1</td>
<td>-18.40</td>
</tr>
<tr>
<td>MMNN (SGA)</td>
<td>25</td>
<td>0</td>
<td>-19.70</td>
</tr>
<tr>
<td>MMNN (BP)</td>
<td>8</td>
<td>1</td>
<td>-19.65</td>
</tr>
<tr>
<td>MMNN (BP)</td>
<td>8</td>
<td>0</td>
<td>-21.10</td>
</tr>
<tr>
<td>MMNN (BP)</td>
<td>25</td>
<td>1</td>
<td>-22.19</td>
</tr>
<tr>
<td>MMNN (BP)</td>
<td>25</td>
<td>0</td>
<td>-23.53</td>
</tr>
<tr>
<td>MMNN (AGA)</td>
<td>8</td>
<td>1</td>
<td>-22.27</td>
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<td>1</td>
<td>-23.11</td>
</tr>
<tr>
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<td>0</td>
<td>-23.76</td>
</tr>
<tr>
<td>MMNN (AGA)</td>
<td>50</td>
<td>1</td>
<td>-23.27</td>
</tr>
<tr>
<td>MMNN (AGA)</td>
<td>50</td>
<td>0</td>
<td>-24.07</td>
</tr>
<tr>
<td>MMNN (AGA)</td>
<td>75</td>
<td>1</td>
<td>-23.31</td>
</tr>
<tr>
<td>MMNN (AGA)</td>
<td>75</td>
<td>0</td>
<td>-24.13</td>
</tr>
<tr>
<td>MMNN (AGA)</td>
<td>100</td>
<td>1</td>
<td>-24.11</td>
</tr>
<tr>
<td>MMNN (AGA)</td>
<td>100</td>
<td>0</td>
<td>-24.27</td>
</tr>
<tr>
<td>MMNN (AGA-MOD)</td>
<td>22</td>
<td>0</td>
<td>-24.32</td>
</tr>
<tr>
<td>AHMOD Method</td>
<td>24</td>
<td>0</td>
<td>-24.81</td>
</tr>
</tbody>
</table>

The AHMOD method automatically chose the MMNN architecture via Matheson Decomposition by dilations \((arch = 0)\) and selected the MMNN modules amount \((mod = 24)\). It is observed that for a noisy density corresponding to 5%, the proposed method presented better performance when compared to other methods presented in the Table II. It is worth to mention that the AHMOD method overperforms the MMNN (AGA) by using a smaller number of MMNN modules. The AHMOD method led to NSR=24.81dB with 24 modules, while MMNN (AGA) led to NSR=24.27dB with 100 modules. The MMNN modules amount decrease is a positive factor when it considers a inquiry of the computational complexity for the morphological operator design. Furthermore, when compared to MMNN (AGA-MOD), the proposed method slightly increased the number of MMNN modules (an increase of two MMNN modules). However, even with a slightly
increase in MMNN modules, the proposed method obtained a gain of 0.29dB. Figure 8 shows the image corresponding to the morphological operator obtained by AHMOD method for Matheron decomposition by 24 dilations.

Fig. 8. Testing images. Results of AHMOD method for Matheron decomposition by 24 dilations. (a) Test image and (b) Restored image.

VII. CONCLUSIONS

An adaptive hybrid morphological method for designing translation invariant operators, via Matheron Decomposition and via Banon and Barrera Decomposition, was presented in this paper. It consists of a hybrid model composed of a Modular Morphological Neural Network (MMNN) and an evolutionary search mechanism: the adaptive Genetic Algorithm (GA) (adaptive rates in genetic operators). The proposed method searches for initial weights, architecture and number of modules in the MMNN; then each element of the adaptive GA population is trained via the Back Propagation (BP) algorithm.

Results have shown that the proposed method is more efficient than the median filter and the methods proposed by Sousa [1], [2] and Araújo et al. [17] for restoring images corrupted by salt and pepper noise. The proposed method has reduced the number of decompositions and the computational complexity in the operators design when compared to MMNN (AGA) [17]. Furthermore, when compared to MMNN (AGA-MOD) [17], the proposed method slightly increased the number of MMNN modules. However, even with a slightly increase in MMNN modules, the proposed method obtained a meaningful gain in terms of noise to signal ratio (NSR).

Future works will consider the proposed method in image segmentation and pattern recognition.

REFERÊNCIAS


