Abstract—Traditional PID controllers have been largely employed for the control of industrial variable speed drives due to easy design and good performance they provide. In this paper, a simple PID (Proportional-Integral-Derivative) speed controller was applied in a switched reluctance motor (SRM) and implemented using dedicated Digital Signal Processor (DSP) with the current control based on Generalized Predictive Control (GPC). To perform the speed controller, the reference current is calculated and applied to SRM drive for obtain speed output. From the experimental results it is shown that the overall switched reluctance motor speed control system gives good transient and the steady state responses and tracks the motion profile given to the system.

I. INTRODUCTION

SWITCHED Reluctance Motors (SRMs) are an alternative and modern solution to electromechanical conversion with variable speed. The availability of high-frequency switching devices and improvements in machine design, associated with SRM intrinsic simplicity, reliability, low costs, high power capacity and fault tolerant operation have made it a viable replacement for conventional motor drives [1], [2]. SRMs have been traditionally controlled by either open loop hysteresis or closed-loop pulse-width-modulation (PWM) current controllers. Each scheme presents advantages and drawbacks with regard to parametric variations, accuracy, robustness, and dynamic response over the entire speed range. The hysteresis current controller is popular because of its inexpensive, simple, and easy-to-use architecture [3]. However, there are well known disadvantages, such as variable switching frequency and high ripple current, making it undesirable for many applications. On the other hand, PWM controllers provide better control loop characteristics compared to their hysteresis counterparts, although they are more complex to be designed and require more computation effort, but such drawbacks can be overcome by using Digital Signal Processors (DSPs). The dynamic setpoint tracking ability and load disturbances recovering without torque ripple are two important challenges for high performance in SRM drives [4]. Several researchers have proposed current profiling based methods to minimize torque ripple [1], [2], [3], [4], [5]. Model-based predictive control (MPC) has also been used in SRM control [6] aggregating a series of advantages over other methods [7]. Within this context, this paper presents a robust control structure with low computational cost based on Generalized Predictive Control (GPC) [8] which belongs to a MPC family. The current and speed controllers are realized using a dedicated DSP designed specifically for motor control. The SRM drive consists on a faster inner current loop based on GPC and an outer PID speed controller. The tuning of the loops is based on a standard response of current and speed of SRM drive. The performance of the motor controller is tested with external load changes. The designed algorithm was implemented in a prototype developed in Electrical Engineering Department of Federal University of Ceará.

The paper is then organized as follows: section II presents an overview of modeling and identification of current loop and of speed loop for SRM drive. A review on GPC is introduced in section III. Section IV describes the proposed current controller. Section V describes the PID controller design for speed loop. Section VI shows the stability analysis and robustness to current loop. Section VII presents experimental results, while the relevant conclusions are discussed in Section VIII.

II. IDENTIFICATION OF SPEED AND CURRENT LOOP TO THE SRM DRIVE

A control block diagram of SRM drive system is shown in Fig. 1 with a control scheme, where an asymmetrical bridge converter topology and PID/PI controller for speed and current of SRM drive has been chosen.

A. SRM Drive Current Loop Identification

In a typical SRM control scheme, the torque is often controlled via an inner current loop. In order to achieve high torque performance, an accurate current tracking is mandatory. The model transfer function given in Eq. (1) can be represented
in a general form as:
\[
G_n(s) = \frac{K_p}{\tau s + 1},
\]  
(1) where \(K_p\) is the gain of the plant and \(\tau\) is the time constant.

The procedure to determine \(K_p\) and \(\tau\) is not simple when no information is provided by the manufacturer or even the machine dimensions are unknown. One way to determine such parameters is through the machine experimentation or complex computational simulations. Another alternative is to estimate them using system identification without disturbing the regular machine operation. A practical identification based on setpoint relay was implemented in [9]. A three-phase SRM was used, whose characteristics are: 12/8, 80 Vdc, 3.5 A, series resistance \(r_s = 2.4 \Omega\), rotor speed at 400 rpm. The inductance varies from a minimum value \(L_u = 8 \text{mH}\) to a maximum one \(L_a = 52 \text{mH}\), what occurs at every 45 mechanical degrees.

As a result of this identification procedure, the transfer function of SRM loop current is given by:
\[
G(z) = \frac{0.03259 z^{-1}}{1 - 0.996 z^{-1}} \approx \frac{0.03259 z^{-1}}{1 - z^{-1}}
\]  
(2)

B. SRM Drive Speed Loop Identification

The speed controller has an inner current control loop and an outer speed control loop. The speed controller generates a reference current based on the error between the reference speed and actual speed. The current in the designated phase is regulated at the reference level by the current controller.

For identification of the SRM speed loop it is necessary to define the current controller that will be used and should be considered the influence of the current loop in the motor speed. Starting from the results presented in the section III the speed loop identification method was accomplished using a PRBS (Pseudo-Random Binary Sequence) between 2 A and 4 A applied to the current loop and the least square method (RLS) [4] was carried out. From the data obtained the discrete transfer function relating speed output and reference current input is given by:
\[
G_v(z^{-1}) = \frac{0.6011 z^{-1} + 0.5905}{1 - 0.3903 z^{-1} - 0.04382 z^{-2}}
\]  
(3)

This approximation allows simplifying the controller design when intended for embedded systems.

III. DESIGN OF CURRENT CONTROLLER USING GPC APPROACH TO MODEL PREDICTIVE CONTROL

The GPC algorithm consists in applying a control sequence that minimizes a multistage cost function of the form [8]:
\[
J = \sum_{j=N_1}^{N_2} [y(t+j|t) - w(t+j)]^2 + \lambda \sum_{j=0}^{N_u-1} |\Delta u(t+j|t)|^2
\]  
(4)

where \(N_1\) and \(N_2\) are the minimum and maximum costing horizons, respectively, \(N_u\) is the control horizon, \(w(t+j)\) is a future setpoint or reference sequence, \(u(t)\) is the incremental control action (\(\Delta = 1 - q^{-1}\); with \(q^{-1}\) being the backward shift operator) and \(y(t+j|t)\) is the optimum \(j\)-step ahead prediction of the system output \(y(t)\) on data up to time \(t\).

The process dynamics can be represented using the Controlled Auto-Regressive and Integrated Moving Average (CARIMA) model [8]:
\[
A(q^{-1}) y(t) = B(q^{-1}) u(t) + C(q^{-1}) e(t),
\]  
(5)

where \(e(t)\) is uncorrelated (white) noise with zero mean value, \(A(q^{-1})\), \(B(q^{-1})\) and \(C(q^{-1})\) are polynomials in the backward shift operator \(q^{-1}\).

The solution of optimization problem of GPC can be done analytically using the predictions \(y(t+j|t)\) and cost function Eq. (4) the prediction output can be written as [11]:
\[
y(t+j|t) = \frac{F_j(q^{-1})}{C(q^{-1})} y(t) + \frac{F_j(q^{-1})}{C(q^{-1})} \Delta u(t-1) + H_j(q^{-1}) \Delta u(t+j-1|t),
\]  
(6)

which can be expressed in a vector form as:
\[
y = F(q^{-1}) \frac{y(t)}{C(q^{-1})} + I(q^{-1}) \frac{\Delta u(t-1)}{C(q^{-1})} + G \Delta u,
\]  
(7)

where:
\[
y = [y(t+N_1|t) \ y(t+N_1+1|t) \ ... \ y(t+N_2|t)]^T,
\]
\[
\Delta u = [\Delta u(t|t) \ u(t+1|t) \ ... \ u(t+N_u-1|t)]^T
\]
and \(G\) is a \(N \times N_u\) constant matrix based on the coefficients of \(H_j(q^{-1})\), while \(F(q^{-1})\) and \(I(q^{-1})\) are polynomial vectors.

From controller implementation standpoint, an analytical solution with low computational cost is important. Thus, this work is concerned with the investigation of a special case where \(N_u = 1, N_1 = 1, N_2 = N\) and \(\lambda = 0\), which represents the best tradeoff between the computational cost and close-loop performance, then the optimal input is [12]:
\[
\Delta u(t) = (G^T G)^{-1} G^T (w - f) = k(w - f),
\]  
(8)

where \(k\) is a constant vector with dimension \(1 \times N\), \(w\) is a vector which contains the future reference and \(f = F(q^{-1}) \frac{y(t)}{C(q^{-1})} + I(q^{-1}) \frac{\Delta u(t-1)}{C(q^{-1})}\) (or free response).

Since \(N_u = 1\), it is important to notice that the constrained controller is equivalent to clipping, a case valid only for monovariable systems [12]. The term clipping assumes that the predictive controller does not take into account constraints while computing the optimal input, but only afterwards, performing hard limitations if constraints are violated.

Through some manipulations, Eq. (8) can be written in the RST form:
\[
u(t) = \frac{1}{\Delta R(q^{-1})} (T(q^{-1}) r(t) - S(q^{-1}) y(t)),
\]  
(9)
where
\[ r(t) = w(t + j) \] (10)
is the setpoint,
\[ T(q^{-1}) = C(q^{-1}) \sum_{i=1}^{N} k(i), \] (11)
\[ S(q^{-1}) = \sum_{i=1}^{N} k(i) F_i(q^{-1}) \] (12)
and
\[ R(q^{-1}) = C(q^{-1}) + q^{-1} \sum_{i=1}^{N} k(i) I_i(q^{-1}). \] (13)

The RST structure is important from control analysis standpoint because it can be derived properties such as stability and robustness.

IV. ROBUST GPC-BASED CONTROL (GPCBC) APPLIED TO SRM CURRENT LOOP

The model which relates the duty cycle to the SRM current loop is given by:
\[ (1 - q^{-1})y(t) = b_0 u(t - 1) + \frac{C(q^{-1})}{\Delta} e(t), \] (14)
where \( b_0 \) is a constant related to the velocity gain of the plant and \( C(q^{-1}) \) is a monic polynomial that can be treated as a filter [11]. The filter design for disturbance rejection and noise attenuation depend on \( C(q^{-1}) \), as the tuning of the related parameters is crucial and was developed in [13].

In this study the process is of first-order and integrative nature, then it is enough to use a filter with degree \( nc = 2 \) in order to attenuate the noise, considering that it is properly tuned [11]. Thus, the proposed filter is given by:
\[ C(q^{-1}) = 1 + c_1 q^{-1} + c_2 q^{-2}, \] (15)
where \( c_1 \) and \( c_2 \) are constants which must be tuned considering noise attenuation, disturbance rejection, and robustness.

Considering Eqs. (14) and (15), the control input \( u(t) \) in Eq. (9) can be calculated explicitly by performing some mathematical manipulation. Thus, the control polynomials \( R, S, \) and \( T \) are given by:
\[ T(q^{-1}) = \frac{(1 - \alpha) C(q^{-1})}{b_0}, \] (16)
\[ R(q^{-1}) = 1 - \alpha c_2 q^{-1}, \] (17)
\[ S(q^{-1}) = \frac{2 - \alpha + c_1 + c_2 - (1 + \alpha c_1 + (2 \alpha - 1) c_2)}{b_0} q^{-1}, \] (18)
\[ \alpha = 1 - \frac{1 + 2 + 3 + \ldots + N}{1 + 2^2 + 3^2 + \ldots + N^2}. \] (19)

It is important to notice that polynomials \( R, S \) and \( T \) contain parameter \( \alpha \), which on the other hand depends on \( N \). From Eq. (19), it can be seen that \( \alpha \) varies from 0 to 1 when the prediction horizon \( N \) varies from 1 to \( \infty \). If \( N \) is used as a tuning parameter, then \( \alpha \) will have discrete values thus making precise tuning impossible. To overcome this problem, the use of \( \alpha \) as a direct tuning parameter is proposed.

The block diagram of the proposed Robust GPC-Based Control (GPCBC) can be posed in the classical RST structure as illustrated in Fig. 2.

This work assumes that \( C(q^{-1}) \) has roots with the same real part, while Eq. (15) can be rewritten as:
\[ C(q^{-1}) = (1 - e^{-\sigma + i\beta} q^{-1})(1 - e^{-\sigma - i\beta} q^{-1}), \] (20)
where \( \sigma \) and \( \beta \) are tuning parameters, and \( i \) is the imaginary operator.

The ratio \( \beta/\sigma \) imposes certain characteristics to the filter \( C \), and therefore a set of filters with different ratios \( \beta/\sigma \) [13].

V. DESIGN PID CONTROLLER FOR SPEED LOOP

The speed loop design considered was the PID control law described in [14]:
\[ \Delta u(k) = K_c \left( 1 + \frac{T_s}{T_i} + \frac{T_d}{T_s} \right) e(k) - K_c \left( 1 + 2 \frac{T_d}{T_s} \right) e(k - 1) + K_c \left( \frac{T_d}{T_s} \right) e(k - 2) \] (21)
where \( e(k) = w(t) - y(t) \), \( w(t) \) is the reference signal and is given by piecewise constant components: \( K_c \), \( T_i \), and \( T_d \) are the proportional gain, the reset time and the derivative time, respectively. Moreover, \( T_s \) denotes the sampling interval. The tuning parameters PID controllers were accomplished using Ziegler-Nichols where the values obtained were:
\[ K_c = 0.3390 \] (22)
\[ T_i = 0.0662 \] (23)
\[ T_d = 0.01587 \] (24)

The PID controller was implemented using the RST structure where described by the following polynomials:
\[ R(z^{-1}) = 1 \] (25)
\[ S(z^{-1}) = s_0 + s_1 z^{-1} + s_2 z^{-2} \] (26)
\[ T(z^{-1}) = S(z^{-1}) \] (27)

Using Eq (21) we have,
\[ s_0 = K_c \left( 1 + \frac{T_s}{T_i} + \frac{T_d}{T_s} \right) \]  \hspace{1cm} (28)
\[ s_1 = -K_c \left( 1 + 2 \frac{T_d}{T_s} \right) \]  \hspace{1cm} (29)
\[ s_2 = K_c \left( \frac{T_d}{T_s} \right) \]  \hspace{1cm} (30)

VI. ROBUSTNESS AND STABILITY ANALYSIS

The robustness analysis is performed considering that the modeling errors of the SRM drive can be represented as unstructured uncertainties, that is: \( G(z) = G_n(z) + \Delta G(z) = G_n(z)(1 + \delta G(z)) \) where \( G_n \) is the nominal plant. Also, let us consider that an upper bound for the norm of \( \delta G(e^{i\Omega}) \) is given by \( \delta G(\Omega) \) for \( 0 \leq \Omega < \pi \) [15].

In order to maintain robust closed-loop stability, the following statement must be satisfied [16]:
\[ \frac{|C(e^{-\alpha i\Omega})(1-\alpha e^{-i\Omega})|}{|S(e^{-\alpha i\Omega})b_o e^{-\alpha i\Omega}|} \leq I_r(\Omega), \]  \hspace{1cm} (31)
for \( \Omega \in [0, \pi] \), and \( I_r(\Omega) \) is defined as the robustness index of the controller. It can be observed that \( I_r(\Omega) \) depends on \( \alpha \) and \( C(e^{-i\Omega}) \), where \( C(e^{-i\Omega}) \) is a tuning factor for robustness [13].

Fig. 3 illustrates the robustness index \( I_r \) of the proposed controllers and the upper bound of the multiplicative error \( \delta G \), which considers \( \pm 10\% \) uncertainties on velocity gain \( (b_o) \) and two samples for time delay.

If disturbance rejection and noise attenuation are the sole criteria to design the filter \( C \), then the best option is to use the filter \( C = C_{75} \), which corresponds to relation \( \beta/\sigma = \tan(75^\circ) \) conform presented in [13], robustness must be also taken into account in order to obtain a final decision.

At low frequencies, it is observed that the robustness index of all controllers is the same; at mid-frequencies, the controllers with larger \( \beta/\sigma \) ratios are less robust, specially \( C_{75} \) which is fair less robust than the remaining filters; at high frequencies, all the controllers have high robustness, particularly those with larger \( \beta/\sigma \) ratio [13].

For a general analysis, the results from the previous section are considered. Besides, it has been shown that the best controller for disturbance rejection and noise attenuation is the one with the larger \( \beta/\sigma \) ratio. Thus, by using the robustness analysis, the controller \( C_{45} \) can be considered as the best choice.

VII. EXPERIMENTAL AND DISCUSSIONS

To explore the effectiveness of proposed controller, experimental tests have been carried out. The proposed controller GPCBC was compared with both a simplified GPC (SGPC) and a conventional PID controller. The controllers have been implemented in a prototype using a digital signal processor TMS320F28335 (DSP), and a classical asymmetric bridge converter, with switching frequency of 25kHz. The experimental setup is presented in Fig. 4.

The plant model used by the current controller is described by Eq. (2). The proposed controller (GPCBC) parameters were chosen as \( \alpha = 0.5 \) to achieve the desired tracking reference and \( \sigma = 0.3 \) with \( C = C_{45} \) \( C(q^{-1}) = 1 - 1.42q^{-1} + 0.55q^{-2} \) to achieve the desired robustness and noise rejection. The SGPC with \( C = 1 \) has a single degree of freedom, while considering \( \alpha = 0.5 \) gives a very low robustness index. The parameter is then set as \( \alpha = 0.8 \) so that similar robustness as that for GPCBC is achieved.

The plant model for the speed controller design was described by Eq. (3). The sampling time are 40 \( \mu \)s and 40 ms for current and speed loop, respectively. To validate this approach in order to demonstrate the usefulness of the proposed method, experimental tests were performed considering the rotor speed varying from 500 rpm to 750 rpm, respectively. Fig. 5 shows the speed response in the no load case under a reference change. The output overshoot at the reference speed of 500 rpm was 31.6% while at the reference speed of 750 rpm was of 8.6% demonstrating the nonlinear behavior of this plant. The settling time was 1.4 s. The corresponding control variable is shown in Fig. 6. The current behavior of this test is shown in Fig. 7. The measured average overshoot was of 4% and a settling time of 480 \( \mu \)s.

The Fig. 8 shows the speed response under a load step is applied during 500 rpm speed operation. The system recovers from the load disturbance in just 0.8 s. Fig. 9 shows the control
variable behavior to this situation. As can be seen in Fig. 5 and 8 the motor speed output follows the reference in steady state and shows robust response regarding load disturbance.

![Fig. 5 Speed response and control variable with change reference, 500 to 750 rpm.](image)

**Fig. 5.** Speed response and control variable with change reference, 500 to 750 rpm.

**Fig. 6.** Control variable with change reference, 500 to 750 rpm.

**Fig. 7.** Phases current with change reference, 500 to 750 rpm.

**Fig. 8.** Speed response to a load step.

**Fig. 9.** Control variable to a load step.

**VIII. CONCLUSION**

A robust control based on model predictive control has been successfully applied to the current control and speed loop of a SRM drive. The approaches has proven to be suitable for SRM drive applications in terms of the achievable control performance, robustness against model mismatches, and unexpected dynamic behavior, as well as in terms of low online computation burden, being suitable for practical use in a wide variety of embedded systems. The tuning parameters controllers using method of GPC and PID based on Ziegler-Nichols presented good results, even out of the operation point initially specified. The current and speed controller was carried out on the TMS320F28335. It is proved that the real time processing capability of the proposed motor control device allows a highly reliable and effective drive with no consideration on system delay and offers a flexibility to implement advanced
or complex control schemes. Experimental results have also demonstrated the feasibility of the proposed SRM drive control schemes.

REFERENCES


